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EDITORIAL.

Proofs of the Pythagorean Theorem

Mr. Arthur R. Colburn, an attorney of Washington, D. C., has taken as his recreation the study of the Pythagorean theorem and its proofs. He started with a knowledge of but two proofs, those of Pythagoras and Euclid, and has rediscovered a good share of the well-known proofs, and a number of others, some of which are little known, if not entirely new.

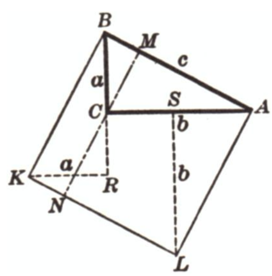
Mr. Colburn claims to have seventy-three proofs, but this number is excessive because some of his proofs are duplicates, in whole or in part, of one or more of the others, or are such that if they are reduced to the essential minimum they become duplicates. Despite this fact, his work might well act as a spur to many teachers who seem to consider geometry as a subject allowing little chance for investigation, owing to its centuries of study by master minds. Even our best teachers might well profit by spending some time in such examination of theorems and the methods of discovering their proofs, for after all, the methods of attack, and the development of original ideas are the great aim of geometrical thought, and unless one is himself in this habit of mind, it is doubtful whether he can give it to others.

One of the most interesting parts of Mr. Colburn's work is a theorem that he states and proves, and by which he finds a number of simple proofs for the Pythagorean theorem. The

theorem is not commonly known, and may be new in this form, although such a claim is hard to establish. It is as follows:

If from one end of a side of a parallelogram a line is drawn to the opposite side (or its extension), and from the other end of that side a perpendicular is drawn to the new line (or its extension), the parallelogram is equivalent to the rectangle of those lines.

The proof is simple, using the fact that parallelograms having equal bases and equal altitudes are equivalent, applied twice, and its application to the Pythagorean theorem is instant, varied, and interesting. The following illustration will show its use:



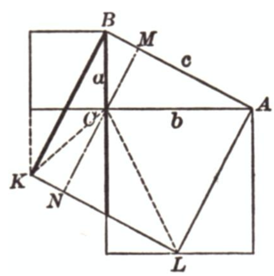
The right triangle ABC is given; the square $ABKL$ is constructed on AB, and MN is drawn perpendicular to AB through C. Then BC (or a), and CA (or b), are lines from a vertex of a parallelogram to the opposite side, so by Colburn's theorem $MBKN$ and $AMNL$ are respectively equivalent to the rectangles of these lines by the perpendiculars to them from K and from L. But these perpendiculars, KR and LS, are easily shown by the triangles BRK and ASL to equal a and b respectively. Therefore $MBKN$ is equivalent to the square on a, and $AMNL$ to the square on b, and the theorem is proved.

Whether this theorem has any other applications that are of value in simplifying the subject is doubtful, although it can be applied in other proofs dealing with squares; for example, it will prove, by equivalence instead of proportion, that the square on a leg of a right triangle is equivalent to the rectangle of the hypotenuse by the projection of the leg on it. The theorem is itself a special case of a more general theorem, which does not seem, after a brief examination, to have any particular value.

In dealing with proofs of the Pythagorean theorem it is difficult to decide how small a variation constitutes a difference in

proof. There can be no doubt that some of the best known proofs are variations of each other, rather than distinct proofs, and it would be an interesting investigation to examine fully all possible methods of proof, grouping them by the principles used. For example, any one of the ordinary equivalence methods that uses the three squares can be proved with six different figures; the method of proof remaining unchanged, as follows:

1. With the square on the hypotenuse outside the triangle, and
   (*a*) the squares on both legs outside,
   (*b*) the square on one leg inside, the other outside,
   (*c*) the squares on both legs inside.
2. With the square on the hypotenuse inside the triangle, and
   (*a*) the squares on both legs outside,
   (*b*) the square on one leg inside, the other outside,
   (*c*) the squares on both legs inside.



For example using 2, (*a*) the Euclidean proof would become, briefly, $BCK = \frac{1}{2}$ square on a, and equals $\frac{1}{2}$ $MBKN$, $ACL = \frac{1}{2}$ square on b, and equals $\frac{1}{2}$ $AMNL$, therefore $ABKL$ (or square on c) is equivalent to square on a plus square on b. In this figure the two triangles that are usually proved congruent have become identical.

Furthermore, the proofs that use the square on the sum of the legs, or any part of that figure, could be grouped, and all possibilities, both of arrangement of parts within the figure, and of omission of various parts not essential to a proof could be listed. If each principle of plane geometry that could apply were thus examined, with all such possibilities for each being considered, the ground would be completely covered. Among the unusual ways of proving this theorem, that, though of no practical importance, may be of interest, might be mentioned the use

of the following propositions, all of which can be proved without its use.

The sum of the squares on two sides of a triangle equals twice the square on half the third side plus twice the square on the median to that side.

The sum of the squares on the sides of a parallelogram equals the sum of the squares on the diagonals.

The sum of the products of the opposite sides of an inscribed quadrilateral equals the product of its diagonals.

The product of two sides of an inscribed triangle equals the product of the altitude to the third side by the diameter.

The product of two sides of a triangle equals the square of the bisector of the included angle plus the product of the sects of the third side.

The wonderful possibilities of this theorem, when examined in a logical way, ought to impress us with the almost infinite richness of geometry; the possibilities of the subject multiply almost to infinity, and the teacher who cannot find material to interest himself and his class in investigations of some such nature is blind to his opportunities.

The Mathematics Section of the Bibliography of Science Teaching

This bibliography was compiled by a committee of the American Federation of Teachers of the Mathematical and the Natural Sciences, and was published by the United States Bureau of Education. While the report lists some excellent works on the elementary and secondary mathematics, it seems a pity that the opportunity was not taken to furnish the teacher with a broader knowledge of recent important committee reports, articles bearing on both sides of the various movements that have aroused discussion, the periodicals that are publishing mathematical subject matter, and, in fact, all that the live teacher would find of interest.

The point most likely to be criticized is that almost all of the works published in the United States that are listed are written by men centered around two of our important universities, and while these works are probably all of sufficient importance to justify their being included, others that many would consider of at least equal importance are excluded. It does not seem

reasonable that there can be such a dearth of material throughout all the rest of our educational system.

Furthermore, the reports of certain associations are included, while other reports that have had at least as strong an influence do not appear. Reports from the American Mathematical Association, and from the Central Association are on the list, as they should be, but why are those of the New England Association committees on arithmetic and geometry lacking, and why should the reports of the Middle States and Maryland Association committees on algebra and on marking examination papers be ignored? Why are teachers not given advance notice of the Federation's own committee on geometry as well as of the work of the International Commission on the Teaching of Mathematics? Again, if the bulletins of one state are included, why should those of other states fail to obtain notice? Many teachers outside of New York State would be interested in the definitions laid down by the Board of Regents, others would be glad to examine the content required for mathematics courses in New Jersey high schools, while all would be likely to be interested in knowing exactly how many states have issued definite statements on mathematics, and how the various states compare in their recommendations. Some of the schools outside of its usual territory may not be familiar with the definitions of the College Entrance Examination Board, and so on. Is it not to list just such publications as these, to call attention to many of the interesting articles appearing in the educational magazines, and, in short, to do for the teacher what he would find it hard to do for himself that this committee was appointed? In looking over the other sections of the bibliography, it appears—although a mathematical magazine is hardly in a position to speak with authority on biology, nature study, and other natural sciences—that considerable work of this kind has been done. It seems, however, that the material would have been of much more value to the one using it if it had been indexed by subjects, and perhaps by source, rather than by authors. If room could be found for that also, so much the better.

To sum up, it seems that the report is very incomplete, and that it falls far short of being of such value to the elementary and secondary teacher as its intended importance would seem to justify.

S.